Affine quantisation of the Brans–Dicke theory

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Introduction

After the formulation of general relativity (GR), some modified theories arose in an attempt to explain open problems in cosmology, such as inflation and the observed accelerated expansion. One of the oldest modifications of GR is the Brans-Dicke theory (BDT), in which there is a non-minimal time-dependent coupling of the long-range scalar field with geometry, that is, with gravity. With the assumption that quantum effects cannot be ignored at early stages of the Universe, the quantisation of the classical BDT in its Hamiltonian description is relevant to better undestand this era. In spite of the fact that the BDT is classically no different from GR, the quantum treatment can reveal new dynamics for the primordial Universe. ArXiv:1810.00707

Affine Quantisation

The affine quantisation is a covariant integral method making use of *coherent states* [1], whose definition is connected with the symmetry of the phase-space. We can identify the half-plane for positive variables with the affine group (group of affine transformations). For positive-definite variables, this quantisation is best suited.

The quantisation of a classical function f, with position q and momentum p, is an integral over coherent states given by

$$f(q,p) \;\mapsto\; A_f = \int_{\Pi_+} f(q,p)\,|q,p\rangle\langle q,p|\,\frac{dq dp}{2\pi c_{-1}}\,,$$

where the constant c_{-1} is defined as

$$c_\gamma = c_\gamma(\psi) := \int_0^\infty |\psi(x)|^2\,\frac{dx}{x^{2+\gamma}}\,,$$

and the ψ's are arbitrary normalised vectors providing square-integrability known as *fiducial vectors*.

The quantisation of position q^β (for any β), momentum p and kinetic energy p^2 yields

$$A_{q^\beta} = \frac{c_{\beta-1}}{c_{-1}}\hat{Q}^\beta;\; A_p = \hat{P};\; A_{p^2} = \hat{P}^2 + \frac{c^{(1)}_{-3}}{c_{-1}}\hat{Q}^{-2}\,.$$

The A_{p^2} operator gets a surprising repulsive potential impeding the variable to reach the origin. Its coefficient depends only on fiducial vectors, and we can recover a unique of quantum object if $4c^{(1)}_{-3} \geq 3c_{-1}$ (**no more ordering problem!**).

Frame Equivalence

Carl Brans and Robert Dicke proposed in 1961 a theory of a varying gravitational constant modelled by a scalar field non-minimally coupled to gravity (Jordan frame)

$$\mathcal{L}_G = \sqrt{-g}\left\{\varphi R - \omega\frac{\varphi_{;\rho}\varphi^{;\rho}}{\varphi}\right\}\,.$$

Performing a conformal transformation allows to rewrite the theory with a minimal coupling (Einstein frame). Classically, the two formulations are equivalent. But there is still doubt on the quantum equivalence.

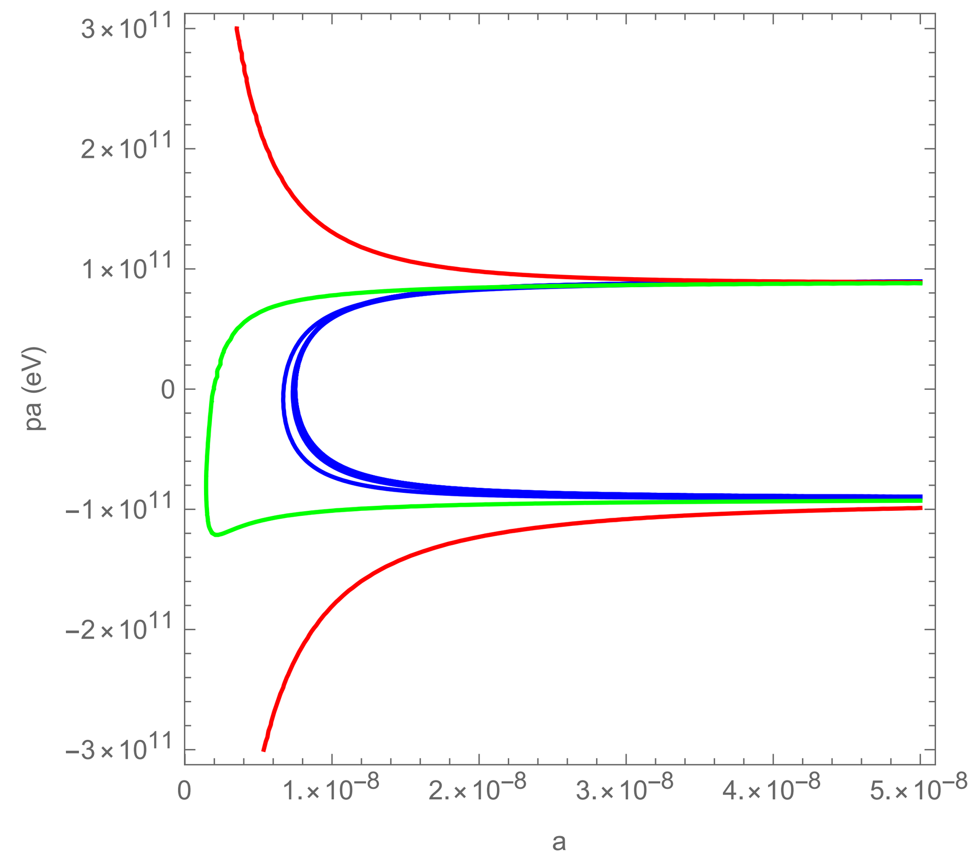
We consider a minisuperspace with the scale factor and the scalar field as variables. Both are always positive hence quantising the Hamiltonian constraint $\mathcal{H} \approx 0$ using the affine quantisation is more suited than the canonical one.

To understand what the difference is between the two frames at the quantum level, we quantise first in the Jordan frame then perform a conformal transformation, and next we perform a conformal transformation followed by the quantisation. All calculations done, this difference amounts to a condition on fiducial vectors

$$c_{-3}(a) = 2\frac{c_{-1}(\varphi)}{c_0(\varphi)}\,.$$

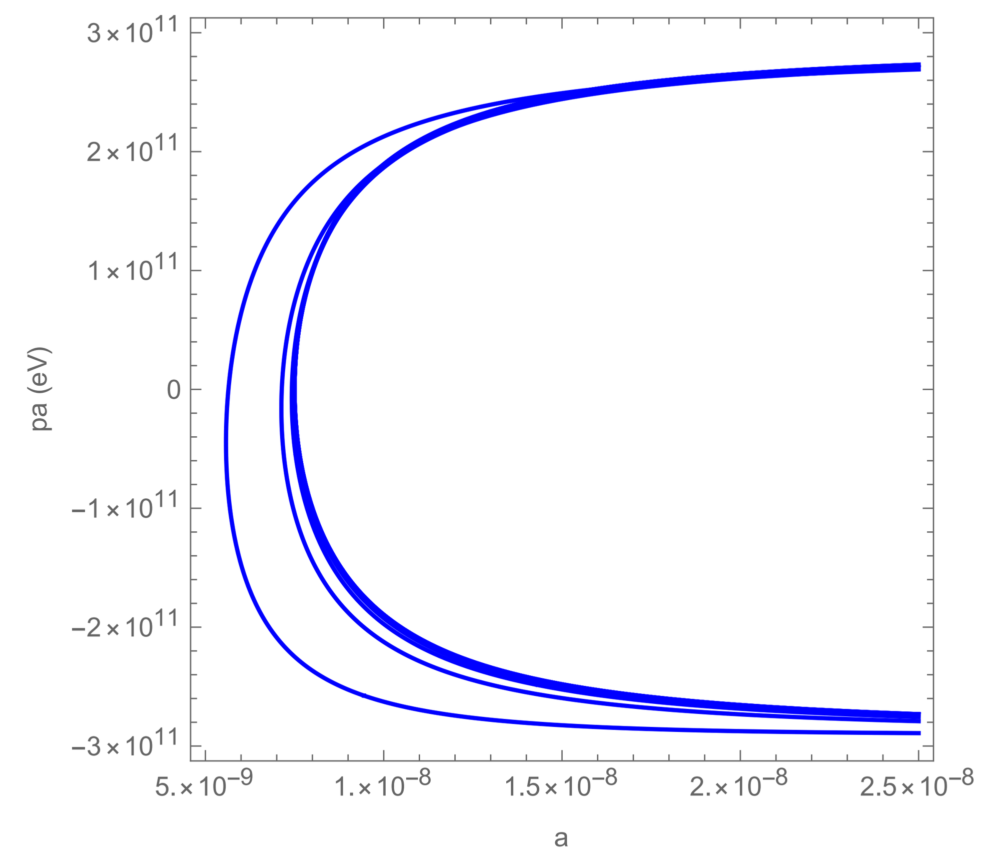
This condition then ensures unitary equivalence between the frames.

Quantum Phase-Space Portraits



The right figure gives the behaviour for $\omega = 4,100,000$. We see the higher values for the momentum no longer give unphysical solutions. The same conclusions are drawn in the Einstein frame, in which the curves are symmetric. Thus, we don't have an inflationary era in this frame. This may be useful to distinguish which frame is the physical one.

The semi-classical limit of the BDT is represented in the Jordan frame. The quantum phase-space of the scalar field, using $\omega = 410,000$ and at some energy scale $E = 10^{16}$, is represented by the left figure for a velocity range $1 \leq p_\varphi \leq 10^3$. The greater p_φ is, the more profound is the bounce, until we reach an upper limit for the field velocity above which the solution is unphysical (red line). Notice the curves are asymmetric, which underlines a change in the momentum of the field around the bounce. We associate this behaviour with an inflationary phase following the bounce.



Conclusion

The affine quantisation is best suited for theories with positive variables. We quantised the scale factor and scalar field in the Brans-Dicke theory, and we concluded that

- an equivalence between the Jordan and Einstein frames at the quantum level depends only on conditions on the fiducial vectors,
- at the semi-classical level, a bounce with a possible inflationary phase in the Jordan frame will occur,
- there is an upper limit for the scalar field velocity.

References

[1] J.P. Gazeau H. Bergeron, A. Dapor and P. Małkiewicz. Smooth Big Bounce from Affine Quantization. *Phys. Rev. D89, 083522*, 13, 2014.

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